International Journal of Theoretical Physics, Vol. 23, No. 12, 1984

Hadron Physics and Transfinite Set Theory

B. W. Augenstein

The Rand Corporation, 1700 Main Street, Santa Monica, California 90406

Received January 4, 1984

Known results in transfinite set theory appear to anticipate many aspects of modern particle physics. Extensive and powerful analogies exist between the very curious theorems on "paradoxical" decompositions in transfinite set theory, and hadron physics with its underlying quark theory. The phenomenon of quark confinement is an example of a topic with a natural explanation via the analogies. Further, every observed strong interaction hadron reaction can be envisaged as a paradoxical decomposition or sequence of paradoxical decompositions. The essential role of non-Abelian groups in both hadron physics and paradoxical decompositions is one mathematical link connecting these two areas. The analogies suggest critical roles in physics for transfinite set theory and nonmeasurable sets.

1. TRANSFINITE SET THEORY AND PHYSICS

Transfinite set theory has a peculiar place in mathematical physics. Modern analysis, on which mathematical physics rests substantially, in turn draws on the abstract theory of sets in many ways—e.g., for the structure of the number system, for measure theory, for set theoretic topology, and for other tools. In this sense, set theory underlies physics. In contrast, the objects of concern to transfinite set theory are not generally considered to be the objects encountered in physics. For this one would have to introduce real infinities into physics, and the implications of such are rarely discussed.

This paper explores connections between the objects of physics and the objects of set theory. Section 2 shows that strong and unexpected analogies exist between hadron physics with its underlying quark theory and certain theorems of transfinite set theory on "paradoxical" decompositions of solid objects in R^3. In effect, hadron physics provides a model satisfying these theorems, if we suppose the particles of hadron physics to be composed of transfinitely many constituents, as is the case for their set theoretic counter-

0020-7748/84/1200-1197$03.50/0

parts (solid objects in R^3). Section 3 includes a table summarizing the analogies, and notes some mathematical implications from the analogies.

The analogies give directly a large number of known physical results (Section 2), and suggest additional ones testable in principle (Section 3). The quark color label and the phenomenon of quark confinement are examples of topics which have immediate explanations via analogies with the decomposition theorems.

Many mathematicians regard "paradoxical" decompositions as rather troubling and abstract curiosities (Section 3). The physicist, familiar with phenomena such as pair creation, no longer considers these phenomena strange. The aspect of "paradoxical" decompositions probably most interesting to the physicist is that these decompositions simulate, at a geometric level, such familiar phenomena of particle physics. The mathematician's "paradoxes" contain an organizing principle for hadron behavior and link transfinite theory with physical objects and events. These links apparently give another remarkable instance of purely geometrical constructs playing an important role in physical laws. Other instances of such roles are noted by Yang (Yang, 1981).

2. PARADOXICAL DECOMPOSITIONS—ROLE IN PARTICLE PHYSICS

Any respectable hadron and quark description should at least account for (i) hadrons being two-quark or three-quark combinations, (ii) inhibitions against hadrons containing more than three quarks, (iii) hadron–quark properties producing quark confinement, and (iv) distinguishable quarks satisfying the exclusion principle.

This paper shows how these and other features of quark theory can be regarded as immediate consequences of a single known existence theorem of transfinite set theory under a suitable, if speculative, interpretation. Additional theorems of this kind lead to other familiar results of hadron physics.

The theorems used are the astonishing paradoxical decomposition theorems which arise in R^m, $m \geqslant 3$. Decomposition involves starting with some initial bounded solid body, partitioning or cutting that solid into a finite number, n, of pieces, and then reassembling those pieces into some final bounded solid or (disjoint) solids. The pieces are to undergo Euclidean transformations only, and so are one-to-one congruent in the sense of elementary geometry in the initial and final solids. The singular aspect of paradoxical decompositions is that volume of the solids need not be conserved in the decomposition and reassembly, although isometry in the